# 1.13 Polar and Euler form

**1a.** *[2 marks]*

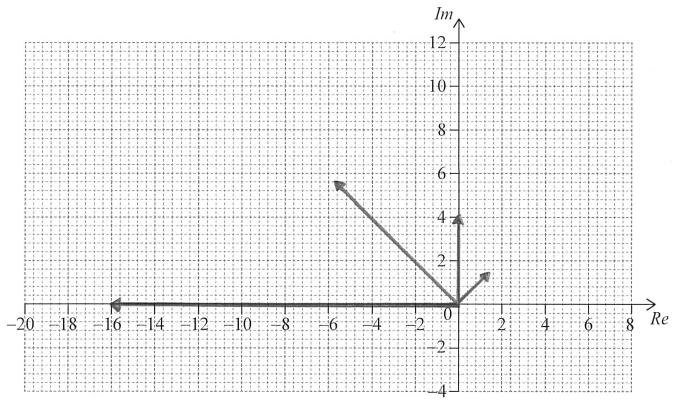
## Markscheme

, ,   (,  ,  ) ***(M1)A1***

***[2 marks]***

**1b.** *[3 marks]*

## Markscheme

 ***A3***

**Note:** Award ***A1*** for correct arguments, award ***A1*** for  and −16 clearly indicated, award ***A1*** for |  | < 4 and 4 < |  | < 16.

***[3 marks]***

**1c.** *[2 marks]*

## Markscheme

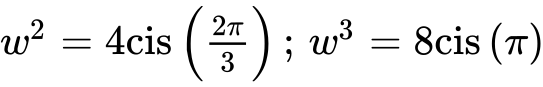
       ***M1***

       ***A1***

***[2 marks]***

**2a.** *[3 marks]*

## Markscheme

     ***(M1)A1A1***

**Note:** Accept Euler form.

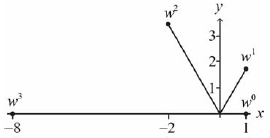
**Note:** ***M1*** can be awarded for either both correct moduli or both correct arguments.

**Note:** Allow multiplication of correct Cartesian form for ***M1***, final answers must be in modulus-argument form.

***[3 marks]***

**2b.** *[2 marks]*

## Markscheme

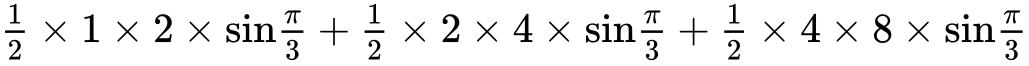
     ***A1A1***

***[2 marks]***

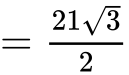
**2c.** *[3 marks]*

## Markscheme

use of area =      ***M1***

      ***A1A1***

**Note:** Award ***A1*** for , ***A1***for correct moduli.

    ***AG***

**Note:** Other methods of splitting the area may receive full marks.

***[3 marks]***

**2d.** *[6 marks]*

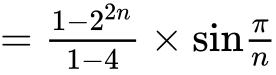
## Markscheme

      ***M1A1***

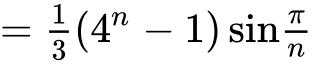
**Note:** Award ***M1***for powers of 2, ***A1***for any correct expression including both the first and last term.



identifying a geometric series with common ratio 2(= 4)     ***(M1)A1***

     ***M1***

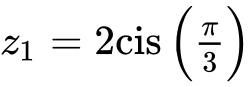
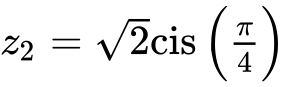
**Note:** Award ***M1*** for use of formula for sum of geometric series.

     ***A1***

***[6 marks]***

**3a.** *[3 marks]*

## Markscheme

 and      ***A1A1***

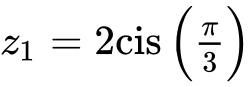
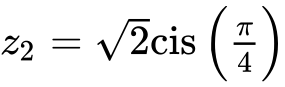
**Note:**     Award ***A1A0*** for correct moduli and arguments found, but not written in mod-arg form.

     ***A1***

***[3 marks]***

**3b.** *[1 mark]*

## Markscheme

 and      ***A1A1***

**Note:**     Award ***A1A0*** for correct moduli and arguments found, but not written in mod-arg form.

     ***A1***

**Notes:**     Allow ***FT*** from incorrect answers for  and  in modulus-argument form.

***[1 mark]***

**3c.** *[2 marks]*

## Markscheme

**EITHER**

     ***(M1)***

**OR**

     ***(M1)***



**THEN**

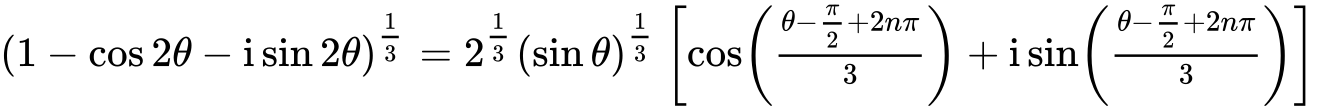
     ***A1***

***[2 marks]***

**4.** *[5 marks]*

## Markscheme

attempt to apply De Moivre’s theorem     ***M1***

     ***A1A1A1***

**Note:**     ***A1*** for modulus, ***A1*** for dividing argument of  by 3 and ***A1*** for .

Hence cube roots are the above expression when . Equivalent forms are acceptable.     ***A1***

***[5 marks]***

**5.** *[4 marks]*

## Markscheme

\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

C represents the complex number      ***A2***

D represents the complex number      ***A2***

***[4 marks]***

**6a.** *[6 marks]*

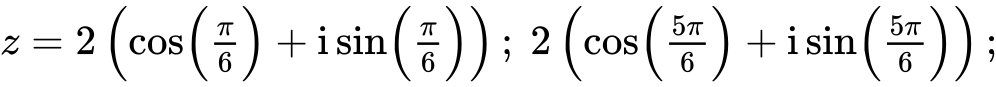
## Markscheme

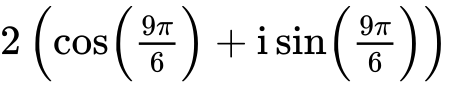
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**Note:** Accept answers and working in degrees, throughout.

     ***(A1)***

attempt the use of De Moivre’s Theorem in reverse     ***M1***



     ***A2***

**Note:** Accept cis form.

     ***A2***

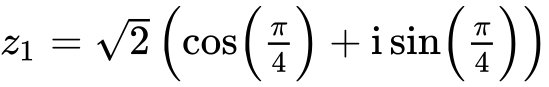
**Note:** Award ***A1*** for two correct solutions in each of the two lines above.

***[6 marks]***

**6b.** *[11 marks]*

## Markscheme

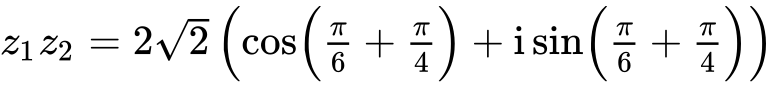
**Note:** Accept answers and working in degrees, throughout.

(i)          ***A1A1***

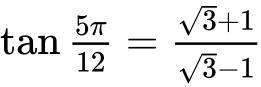
(ii)     

     ***M1***

     ***A1***

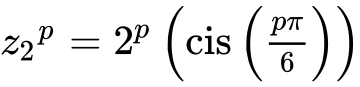
(iii)          ***M1A1***

**Note:** Interpret “hence” as “hence or otherwise”.

     ***A1***

     ***M1A1***

**Note:** Award final ***M1*** for an attempt to rationalise the fraction.

(iv)          ***(M1)***

 is a positive real number when      ***A1***

**Note:** Accept a solution based on part (a).

***[11 marks]***

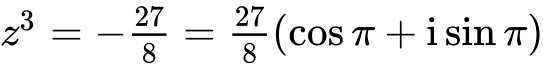
***Total [17 marks]***

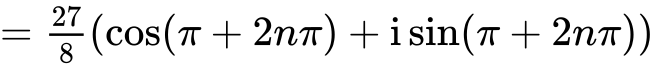
**7a.** *[6 marks]*

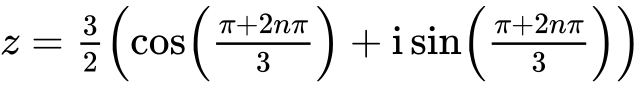
## Markscheme

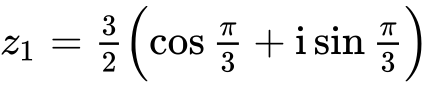
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

**METHOD 1**

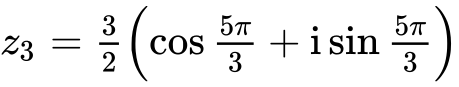
     ***M1(A1)***

     (***A1)***

     ***M1***

,

,

.     ***A2***

**Note:** Accept  as the argument for .

**Note:** Award ***A1*** for  correct roots*.*

**Note:** Allow solutions expressed in Eulerian  form.

**Note:** Allow use of degrees in mod-arg (r-cis) form only.

**METHOD 2**



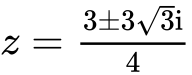
 so  is a factor

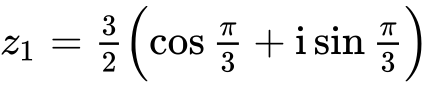
Attempt to use long division or factor theorem:     ***M1***



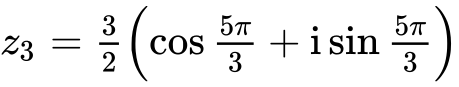
     ***A1***

Attempt to solve quadratic:     ***M1***

     ***A1***

,

,

.     ***A2***

**Note:** Accept  as the argument for .

**Note:** Award ***A1*** for  correct roots*.*

**Note:** Allow solutions expressed in Eulerian  form.

**Note:** Allow use of degrees in mod-arg (r-cis) form only.

**METHOD 3**



Substitute      ***M1***

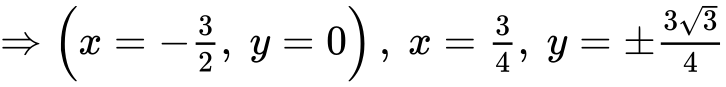


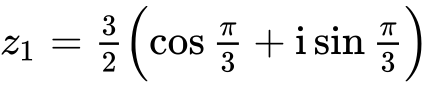
 and      ***A1***

Attempt to solve simultaneously:     ***M1***

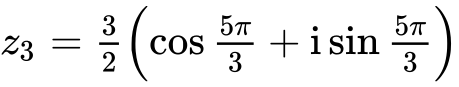




     ***A1***

,

,

.     ***A2***

**Note:** Accept  as the argument for *.*

**Note:** Award ***A1*** for  correct roots*.*

**Note:** Allow solutions expressed in Eulerian  form.

**Note:** Allow use of degrees in mod-arg (r-cis) form only.

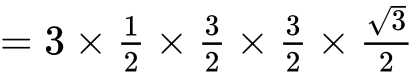
*[6 marks]*

**7b.** *[3 marks]*

## Markscheme

**EITHER**

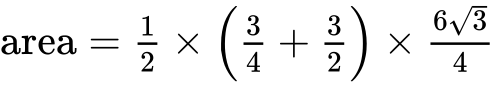
Valid attempt to use      ***M1***

     ***A1A1***

**Note:** Award ***A1*** for correct sides, ***A1*** for correct sin .

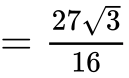
**OR**

Valid attempt to use      ***M1***

     ***A1A1***

**Note:     *A1*** for correct height, ***A1*** for correct base.

**THEN**

     ***AG***

***[3 marks]***

***Total [9 marks]***

**8.** *[17 marks]*

## Markscheme

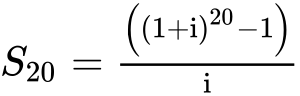
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

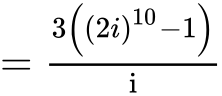
(a)          ***(A1)***

     ***M1***

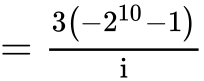
     ***A1***

***[3 marks]***

(b)          ***(M1)***

     ***(M1)***

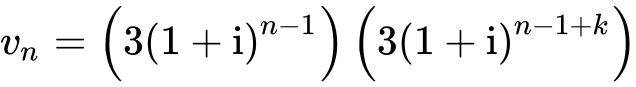
**Note:**     Only one of the two ***M1***s can be implied. Other algebraic methods may be seen.

     ***(A1)***

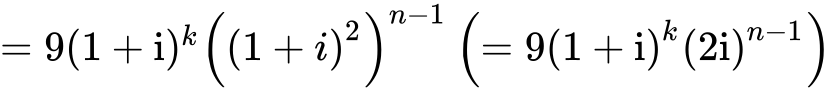
     ***A1***

***[4 marks]***

(c)     (i)     **METHOD 1**

     ***M1***

     ***A1***

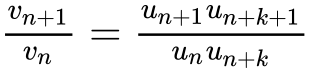


this is the general term of a geometrical sequence     ***R1AG***

**Notes:**     Do not accept the statement that the product of terms in a geometric sequence is also geometric unless justified further.

     If the final expression for  is  award ***M1A1R0***.

**METHOD 2**

     ***M1***

     ***A1***

this is a constant, hence sequence is geometric     ***R1AG***

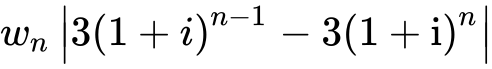
**Note:**     Do not allow methods that do not consider the general term.

(ii)          ***A1***

(iii)     common ratio is  (which is independent of *k*)     ***A1***

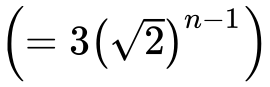
***[5 marks]***

(d)     (i)     **METHOD 1**

     ***M1***

     ***M1***

     ***A1***



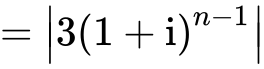
this is the general term for a geometric sequence   ***R1AG***

**METHOD 2**

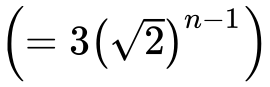
     ***M1***



     ***A1***



     ***A1***



this is the general term for a geometric sequence     ***R1AG***

**Note:**     Do not allow methods that do not consider the general term.

(ii)     distance between successive points representing  in the complex plane forms a geometric sequence     ***R1***

**Note:**     Various possibilities but must mention distance between successive points.

***[5 marks]***

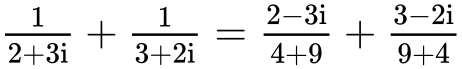
***Total [17 marks]***

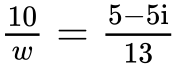
**9.** *[7 marks]*

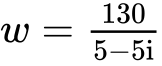
## Markscheme

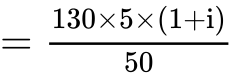
\* This question is from an exam for a previous syllabus, and may contain minor differences in marking or structure.

(a)     **METHOD 1**

     ***M1A1***

     ***A1***

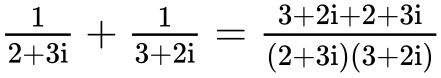


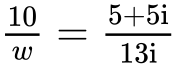


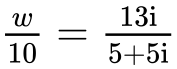
     ***A1***

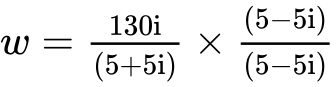
***[4 marks]***

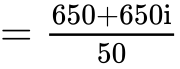
**METHOD 2**

     ***M1A1***

     ***A1***



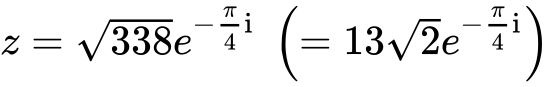


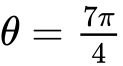


     ***A1***

***[4 marks]***

(b)     *w*\*      ***A1***

     ***A1A1***

**Note:**     Accept .

     Do not accept answers for  given in degrees.

***[3 marks]***

***Total [7 marks]***

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